Regras de Inferência

Dedução Natural

Note que $\neg x \equiv x \rightarrow \perp$

$$\frac{x \qquad y}{x \wedge y}\ (\wedge I) \qquad \frac{x \wedge y}{x}\ (\wedge E_1) \qquad \frac{x \wedge y}{y}\ (\wedge E_2)$$

$$\frac{x}{x \vee y}\ (\vee I_1) \qquad \frac{y}{x \vee y}\ (\vee I_2) \qquad \frac{x \vee y \qquad [x] \vdash z \qquad [y] \vdash z}{z}\ (\vee E)$$

$$\frac{[\neg x] \vdash \perp}{x}\ (RAA) \qquad \frac{x}{x}\ (ID) \qquad \frac{\perp}{x}\ (CTR)$$

$$\frac{x \rightarrow y \qquad x}{y}\ (\rightarrow E) \qquad \frac{[x] \vdash y}{x \rightarrow y}\ (\rightarrow I)$$

$$\frac{P(x) \qquad x \notin fv(\Gamma)}{\forall x.P(x)}\ (\forall I) \qquad \frac{\forall x.P(x)}{P(a)}\ (\forall E)$$

$$\frac{P(b)}{\exists x P(x)}\ (\exists I) \qquad \frac{\exists x P(x) \qquad P(k) \vdash z \qquad k \notin fv(\Gamma)}{z}\ (\exists E)$$

Álgebra Booleana

$\alpha \wedge \bot \equiv \bot$	$\{\wedge -$ null $\}$
$\alpha \vee \top \equiv \top$	$\{\vee -$ null $\}$
$\alpha \wedge \top \equiv \alpha$	$\{\wedge -$ identidade $\}$
$\alpha \vee \bot \equiv \alpha$	$\{\vee -$ identidade $\}$
$\alpha \wedge \alpha \equiv \alpha$	$\{\wedge -$ idempotencia $\}$
$\alpha \vee \alpha \equiv \alpha$	$\{\vee -$ idempotencia $\}$
$\alpha \wedge \beta \equiv \beta \wedge \alpha$	$\{\wedge -$ comutativo $\}$
$\alpha \vee \beta \equiv \beta \vee \alpha$	$\{\vee -$ comutativo $\}$
$\alpha \wedge (\beta \wedge \gamma) \equiv (\alpha \wedge \beta) \wedge \gamma$	$\{\wedge -$ associativo $\}$
$\alpha \vee (\beta \vee \gamma) \equiv (\alpha \vee \beta) \vee \gamma$	$\{\vee -$ associativo $\}$
$\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$	$\{\wedge -$ distribui $-\vee\}$
$\alpha \vee (\beta \wedge \gamma) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$	$\{\vee -$ distribui $-\wedge\}$
$\neg(\alpha \wedge \beta) \equiv \neg\alpha \vee \neg\beta$	$\{$ DeMorgan $-\wedge\}$
$\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$	$\{$ DeMorgan $-\vee\}$
$\neg\top \equiv \bot$	$\{$ negação $-\top\}$
$\neg\bot \equiv \top$	$\{$ negação $-\bot\}$
$\alpha \wedge \neg\alpha \equiv \bot$	$\{$ complemento $-\wedge\}$
$\alpha \vee \neg\alpha \equiv \top$	$\{$ complemento $-\vee\}$
$\neg(\neg\alpha) \equiv \alpha$	$\{$ dupla-negação $\}$
$\alpha \rightarrow \beta \equiv \neg\alpha \vee \beta$	$\{$ implicação $\}$
$\alpha \leftrightarrow \beta \equiv (\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$	$\{$ bicondicional $\}$

$\neg\forall x.P(x) \equiv \exists x.\neg P(x)$	$\{\neg - \forall\}$
$\neg\exists x.P(x) \equiv \forall x.\neg P(x)$	$\{\neg - \exists\}$
$\forall x.P(x) \wedge Q(x) \equiv \forall x.P(x) \wedge \forall x.Q(x)$	$\{\wedge - \forall\}$
$\exists x.P(x) \vee Q(x) \equiv \exists x.P(x) \vee \exists x.Q(x)$	$\{\vee - \exists\}$